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NASA Infrared Telescope Facility

TCS SERVO SYSTEM: Mechanical Modeling



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Overview:

The following is a document describing the work performed with respect to mechanical issues, for the TCS servo design.

Early in the development of the new IRTF telescope control system (TCS) it was determined that we needed to fully define the servo system in order to understand its impact on overall system specifications. How the servo loop effected the specifications would also determine how other contributions to the image degradation (such as wind disturbances, encoder resolutions, nonlinearities...) would be handled. We decided that we needed to construct a simplified analytical mechanical model of the telescope, and to this model, we would apply our new TCS system in order to determine the final performance.

A mathematical model was first constructed, and the governing differential equations were processed for a solution. In the end, the equations became unwieldy and it was decided that we would need to implement a computer mathematical package (Matlab) in order to solve the equations. After starting work in the package, we discovered that there were "built in" blocks and subroutines that could greatly simplify the modeling process for us. In the end, we simply modeled the entire system in Matlab Simulink.

The first step in the process was to reverse engineer a JPL document that was produced in the preliminary stages of the IRTF development (mid 1970's). The information used in that document was then used to construct a model of the IRTF in its current state – with its current analogue control system. Once we were happy with this model, we stripped the analogue control system away, and then began to implement the new digital control system.

Our final model was lacking in any type of external disturbances, and so an effort was undertaken to add wind loading to the structure. It was felt that this would be the most significant external disturbance to the system, and a good test of the controls systems disturbance rejection.

JPL Model (5 DOF):

Background:

See JPL Interoffice Memo 3555-76-031: Dynamic Analysis of the 3M Mauna Kea Telescope, and Interoffice Memo 354-76-761: Preliminary Input Parameters for Servo Analysis Model.

The JPL documents mentioned above, were extremely thorough in their development of an analytical model; however, there were several things that we felt we could / should improve upon.

Firstly, the analytical model incorporated both the mechanical system as well as the control system into one single model. We needed to split the two systems apart in order that we would be able to replace the control portion in the future with our new digital system. The resulting mechanical portion of the model is the subject of this document.

Secondly, we felt that we needed to add an extra degree of freedom to account for the fact that there were two drive motors on the IRTF, each with its own tachometer. The JPL model contained only a single motor with a single tachometer. We felt that this was a significant difference, and that we needed to model the two motor system in order to correctly determine the subtleties of the velocity loop (which ends up being the heart of the disturbance rejection system).

Lastly we decided to add some random noise to the system in the form of wind loading. We felt that it was important to try to add some form of disturbance to the system in order to exercise the velocity loops of the servo.

The 5 DOF Mechanical Model:

A lumped mass model of the telescope was constructed with the following degrees of freedom:

- 1) Telescope foundation.
- 2) South pier, including the motor stator and gear case.
- 3) Motor rotor and drive gears.
- 4) Telescope yoke, tube, and bull gear.
- 5) North pier.

Attached drawing #2401-1 shows the model in Fig. #1, with all of the variables defined. The 5 governing equations for the five degrees of freedom were derived and listed below the model. It should be noted that all of the degrees of freedom are rotational.

In the model, the k_{ij} 's are associated with stiffness (or rotational spring coefficients) between the elements, the J_i 's are the mass moment of inertia of the rotational elements, the b_{ij} 's are the damping coefficients between the elements, and the $Ø_i$'s are the angular position of the element. The values used for these coefficients are taken from the JPL derivation and are listed as follows:

$b_{01} = 35.929 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	$J_1 = 21743 \times 10^3 \text{ kg} \cdot \text{m}^2$
$b_{12} = 0.168 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	$J_2 = 7897 \text{ kg} \cdot \text{m}^2$
$b_{23} = 0.260 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	$J_3 = 32629 \text{ kg} \cdot \text{m}^2$
$b_{34} = 0.088 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	$J_4 = 532716 \text{ kg} \cdot \text{m}^2$
$b_{45} = 0.373 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	$J_5 = 169475 \text{ kg} \cdot \text{m}^2$
$b_{51} = 0.254 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$	
$k_{01} = 4561 \text{ x } 10^6 \text{ N} \cdot \text{m}$	$kk_1 = 2.7397 \times 10^{-3} \text{ N} \cdot \text{m/volt}$
$k_{12} = 8993 \text{ x } 10^6 \text{ N} \cdot \text{m}$	$m = .6044 \text{ x } 10^6 \text{ N} \cdot \text{m} \cdot \text{s}$
$k_{34} = 587 \text{ x } 10^6 \text{ N} \cdot \text{m}$	
$k_{51} = 955 \text{ x } 10^6 \text{ N} \cdot \text{m}$	

Details of how these coefficients are derived are given in the JPL documents.

The 5 DOF MATLAB Model:

From the differential equations, the Matlab model was created. The actual model is shown in the attached drawing #2402-1. The model takes as an input, a single torque (in N·m) and gives an output of position (in rad) of the telescope tube, and velocity (in rad/s) of drive motors. These are actually given relative to the south pier (J_2), just as the actual encoders on the telescope would produce.

Model Verification:

From the model, we are able to run simulations and determine where the "locked rotor" resonant frequencies were. It would be extremely useful if time permitted in the future, to attempt to determine the first few (lowest) natural frequencies on the actual telescope. This would be one method for confirming the assumed spring constants, and mass moments of inertia from the JPL document.

IRTF Model (6 DOF):

Background:

As was mentioned previously, we felt that one of the most important things for us to model was the disturbance rejection of the system, or more precisely - the effectiveness of the velocity loops.

The model developed by the JPL staff contained only one motor with a single tachometer on it. The actual IRTF has two motors back-driving against each other, with an individual tachometer on each motor. We decided that we needed to add this second motor (and tachometer) to our model if we were really serious about using the model to simulate our telescope.

The 6 DOF Mechanical Model:

Again, a lumped mass model of the telescope was constructed, this time with the following degrees of freedom:

- 1) Telescope foundation.
- 2) South pier, including the motor stator and gear case.
- 3) A motor rotor and drive gears.
- 4) B motor rotor and drive gears.
- 5) Telescope yoke, tube, and bull gear.
- 6) North pier.

Attached drawing #2403-1 shows the model in Fig. #1, with all of the variables again defined. The 6 governing equations for the 6 degrees of freedom are derived from the model and listed below it.

In the JPL 5 DOF model, the stiffness and damping constants, associated with the motor rotor and gears, were determined experimentally from a test at KPNO (see JPL documentation). These values were added into the model, and simply applied in a negative sense if the motor was driving in a negative direction (this assumes zero backlash). What this means is that we can use the exact same values in our 6 DOF model, as each motors will only apply a moment in a single direction (opposing each other to remove the backlash).

The model is shown in terms of its spring and damping coefficients, and its mass moments of inertia. Once again, the values used for these coefficients are taken from the JPL derivation and are listed as follows:

$J_1 = 21743 \text{ x } 10^3 \text{ kg} \cdot \text{m}^2$
$\mathbf{J}_2 = 7897 \text{ kg} \cdot \text{m}^2$
$J_{3a} = 32629 \text{ kg} \cdot \text{m}^2$
$J_{3b} = 32629 \text{ kg} \cdot \text{m}^2$
$J_4 = 532716 \text{ kg} \cdot \text{m}^2$
$J_5 = 169475 \text{ kg} \cdot \text{m}^2$
$kk_{1} = 2.7397 \text{ x } 10^{-3} \text{ N} \cdot \text{m/volt}$
$m = 6044 \times 10^6 \text{ N m}^{\circ}$
$III = .0044 \times 10^{-1} IV III S$

Again, details of how these coefficients are derived are given in the JPL documents.

The 6 DOF MATLAB Model:

From the differential equations, the Matlab model was created. The actual model is shown in the attached drawing #2404-1. The model takes as inputs, two torques (in N·m) and gives as outputs, position (in rad) of the telescope tube, and the two velocities (in rad/s) of the two drive motors. These are actually given relative to the south pier (J₂), just as the actual encoders on the telescope would produce.

Model Verification:

Again, from the model, we are able to run simulations and determine where the "locked rotor" resonant frequencies were. It would be extremely useful if time permitted in the future, to attempt to determine the first few (lowest) natural frequencies on the actual telescope. This would be one method for confirming the assumed spring constants, and mass moments of inertia from the JPL document.

Disturbance Rejection:

Wind Loading

Background:

Early in the TCS project, there were questions as to whether a velocity loop was really required in the servo design. Most of the literature studied, seemed to suggest that the velocity loop was imperative in order to meet the disturbance rejection requirements. We felt that in order to fully comprehend the importance of the velocity loops, we should try to add some form of disturbance to our model.

In reality, one of the most significant of these disturbances is "wind loading", and so we decided to add some form of wind loading to our model.

It should be noted that the wind loading model used in the following calculations is by no means the "most correct" model. It is just a simple approximation as to what we might see at the telescope, and is really just used as a "ball park" approximation of a disturbance for testing our servo system's disturbance rejection ability. The weaknesses in our model will be pointed out as we develop it in the following sections.

Steady State Loading:

A calculation was performed to determine the steady state loading on the top end of the telescope structure, due to a wind force.

Our calculation is based on empirical data and is common in any standard fluid dynamics textbook. Details of the calculation are given in drawing #2405-1 and #2405-2. All of the assumptions made are noted in the drawings, and the final result may be summarized as:

The approximate torque applied to the telescope axis due to drag induced by a 10mph wind on the top end structure is determined to be $488.1 \text{ N}\cdot\text{m}$.

The largest assumption made in this approximation is that the top end would be exposed to a free air velocity of 10mph. This is analogous to assuming that the telescope is sitting in a large open field, exposed to the 10mph wind. In reality, our telescope is located inside our dome, and is somewhat sheltered from a much larger (typically 30mph) wind speed outside of the dome.

Wind Power Spectral Density:

In reality, the wind forces that the telescope will be exposed to will be time varying and somewhat erratic. This is a result of the turbulent flow inside the dome as well as the result of vortex shedding off of the structure itself.

Much of the literature found on the subject of wind loading showed PSD curves that rolled off at higher frequencies. As a result we attempted to construct a model based on a "band limited white noise generator". Theoretically, the PSD curve of this signal is a constant and so we simply processed the signal with a Butterworth Low-Pass filter with the desired cutoff frequency. The signal was then given the appropriate gain factor in order to produce an RMS value identical to the steady state value determined above.

The MATLAB Wind Simulation Model:

The Matlab model used for the wind simulation is shown in drawing #2406-1 and the scope results are shown below:



The top trace is the signal passed through a 5 rad/s roll-off filter and the plot immediately below it is of the cumulative RMS value.

The third trace is the signal passed through a 2 rad/s roll-off filter and the plot immediately below it is of the cumulative RMS value.

The fifth trace is the signal passed through a 0.2 rad/s roll-off filter and the plot immediately below it is of the cumulative RMS value.

As can be seen, the peak to valley of the signal is approximately 3 times the RMS value, as would be expected for a normally distributed random variable. The assumption that we have made is that at the telescope, the 10mph value that we used for steady state loading is actually the RMS of what we would measure inside the dome. This may be somewhat conservative, and result in loads that are actually larger than we would see in reality.

The end result is that the signal that is sent to the telescope tube in our telescope model is the signal selected above, normalized to give a RMS value of 500 N·m (as shown by the final gain block in the model).

The three different values selected for roll-off were chosen arbitrarily. In reality, the 5 rad/s roll-off signal would be more difficult for the servo to correct, but the 0.2 rad/s signal seemed to be a more appropriate signal to use "intuitively". The model was left with all three signals in it so that the user could select which one he felt was appropriate.

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$L = kk_1 \cdot e_{1-in}(t) - m(\dot{\theta}_3 - \dot{\theta}_2)$	$J_{1}\dot{\theta}_{1} = -k_{01}\theta_{1} - k_{12}(\theta_{1} - \theta_{2}) - J_{2}\dot{\theta}_{2} = -k_{12}(\theta_{2} - \theta_{1}) - b_{12}(\dot{\theta}_{2} - \beta_{3}) - b_{23}(\dot{\theta}_{2} - \beta_{3}) - b_{23}(\dot{\theta}_{3} - \beta_{4}) - b_{23}(\dot{\theta}_{3} - \beta_{4}) - b_{23}(\dot{\theta}_{3} - \beta_{4}) - b_{34}(\dot{\theta}_{3} - \beta_{5}) - b_{34}(\dot{\theta}_{5} - \beta_{5}) - b_{45}(\dot{\theta}_{5} - \beta_{5}) - b_{4$	bo	1 [Figure #1: Earth Dynamic Model
$\begin{aligned} kk_{1} &= motor\ constant\ (N\cdot m \ Volt) \\ m &= motor\ constant\ (N\cdot m \ s) \\ e_{i-in}(t) &= motor\ input\ voltage\ (Volts) \\ \hline \frac{1}{\text{REV}\ loc} & \frac{1}{\text{DESCRIPTION}} & \frac{1}{\text{BT}} & \frac{1}{\text{DATE}} \\ \hline \\$	$\begin{split} & k_{51}(\theta_1 - \theta_5) - b_{01}\dot{\theta_1} - b_{12}\left(\dot{\theta_1} - \dot{\theta_2}\right) - b_{15}\left(\dot{\theta_1} - \dot{\theta_5}\right) \\ & z_2 - \dot{\theta_1} - b_{23}\left(\dot{\theta_2} - \dot{\theta_3}\right) - L \\ & z_3 - \dot{\theta_2} - b_{34}\left(\dot{\theta_3} - \dot{\theta_4}\right) + L \\ & z_4 - \dot{\theta_3} - b_{45}\left(\dot{\theta_4} - \dot{\theta_5}\right) \\ & z_4 - \dot{\theta_4} - b_{15}\left(\dot{\theta_5} - \dot{\theta_1}\right) \end{split}$	b_{12} Gear Case, b_{23} and South Pier b_{23} and	θ_1 k_{12} θ_2 C Motor Torque
ALL DIRENSIONS IN MULMETERS TOLEWAGES X. TOLEWAGES X. TOLEWAGES HOROLUL, Hawaii X. TOLEWAGES HOROLUL, Hawaii X. TOLEWAGES X. TOLEWAGES HOROLUL, Hawaii X. TOLEWAGES M. TOLEWAGES HOROLUL, Hawaii X. TOLEWAGES M. TOLEYAGES M. TOL	$\begin{split} J_{ij} &= \text{moment of inertia } (\text{kg} \cdot \text{m}^2) \\ b_{ij} &= \text{damping constants } (\text{N} \cdot \text{m} / \text{rad}^2) \\ k_{ij} &= \text{spring constants } (\text{N} \cdot \text{m} / \text{rad}) \\ L_i &= \text{motor torque } (\text{N} \cdot \text{m}) \end{split}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
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$p = 61633 \text{ Pa}$ $\rho = .8191 \text{ kg/m}^3$ $\mu = 1.71 \times 10^{-5} (\text{N} \cdot \text{s})/\text{m}^2$ Absolute viscosity is heavily dependant on temperature, and mildly dependant on pressure. We are using a value for air at 0°C and 1 atm. $\frac{\frac{1}{\text{NEVISION}} + \frac{\text{REVISION}}{\frac{\text{REVISION}}{\text{REVISION}} + \frac{1}{3} + \frac{1}{3$	The Reynold's number is calculated as follows: $ \begin{aligned} \rho &= \text{density} \begin{bmatrix} \text{kg/m}^3 \end{bmatrix} \\ V &= \text{velocity} \begin{bmatrix} \text{m/s}^2 \end{bmatrix} \\ 1 &= \text{characteristic length} \begin{bmatrix} \text{m} \end{bmatrix} \\ \mu &= \text{absolute viscosity} \begin{bmatrix} \frac{N \cdot \text{s}}{\text{m}^2} \end{bmatrix} \end{aligned} $ At an altitude of 4000m, 0°C, air has the following properties:	 (2) The properties of air were calculated at 4000m altitude and 0° Celcius. The telescope structure was approximated as a series of circular and rectangular tubes. Only the upper trusses and secondary rings were considered in the calculation. This approximates a "near zenith" orientation of the telescope. 	1 2 3 The following is a sample calculation used to determine the steady state component of the force applied to the upper telescope structure due to wind loading. Several assumptions made are as follows: 1 The wind velocity that the calculation is performed at is 10 mph. This is the open air velocity (velocity before and after the example, a 30 mph wind outside the dome may approximate to a 10 mph wind inside the dome. 3
MLESS Offermise syntps AL DMESSION IN MILLIMETERS NASA INFARED TELESCOPE FACILITY Institute for Astronomy Institute for Astronomy XX ± 0.000 Institute for Astronomy XXX ± 0.000 Prover MICE ± 0.000 Prover Institute for Astronomy Description MARK ± 0.000 Prover MARK ± 0.000 Prover Instructions Prover Prover Instructions Wind Loading Calculations Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions Instructions <t< td=""><td>Where the drag coefficients for several different geometries are given below: $\square \qquad C_{\rm p} = 2.1 \text{ (Square Cylinder)}$ $\square \qquad \bigcirc \qquad C_{\rm p} = 1.2 \text{ (Round Cylinder, } 10^{+} < R_{e} < 2 \times 10^{5}\text{)}$ $\square \qquad \bigcirc \qquad C_{\rm p} = 0.3 \text{ (Round Cylinder, } R_{e} > 2 \times 10^{5}\text{)}$</td><td>$R_{e} = \frac{(.8191)(4.47)(.15)}{(1.71 \times 10^{-5})} = 32.12 \times 10^{3}$ Based on experimental data for Re# > 10000, we can use the following equation for an approximation of the drag: $drag = C_{b} \left(\frac{1}{2}\rho V^{2}A\right) (EQ2) \qquad \begin{array}{c} A = Frontal Area (m^{2}) \\ C_{D} = Drag Coefficien t \end{array}$</td><td>$4 5 6$ $\frac{\text{BLL OF MATERIAL}}{\frac{\text{mem}}{-} 6}$ SUMMIT WIND = $(10\text{mph})\left(\frac{1.6093}{3.6}\right)$ = 4.47 m/s The characteristic length that we will use is 0.15m. This is the approximate width of the trusses and rings that we are determining the drag for.</td></t<>	Where the drag coefficients for several different geometries are given below: $\square \qquad C_{\rm p} = 2.1 \text{ (Square Cylinder)}$ $\square \qquad \bigcirc \qquad C_{\rm p} = 1.2 \text{ (Round Cylinder, } 10^{+} < R_{e} < 2 \times 10^{5}\text{)}$ $\square \qquad \bigcirc \qquad C_{\rm p} = 0.3 \text{ (Round Cylinder, } R_{e} > 2 \times 10^{5}\text{)}$	$R_{e} = \frac{(.8191)(4.47)(.15)}{(1.71 \times 10^{-5})} = 32.12 \times 10^{3}$ Based on experimental data for Re# > 10000, we can use the following equation for an approximation of the drag: $drag = C_{b} \left(\frac{1}{2}\rho V^{2}A\right) (EQ2) \qquad \begin{array}{c} A = Frontal Area (m^{2}) \\ C_{D} = Drag Coefficien t \end{array}$	$4 5 6$ $\frac{\text{BLL OF MATERIAL}}{\frac{\text{mem}}{-} 6}$ SUMMIT WIND = $(10\text{mph})\left(\frac{1.6093}{3.6}\right)$ = 4.47 m/s The characteristic length that we will use is 0.15m. This is the approximate width of the trusses and rings that we are determining the drag for.



