Minimum energy catastrophic disruptions

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Angular Momentum of Small Bodies

• The rotational angular momentum of small bodies are not constant over time, due to:
  – The YORP effect
    • Sunlight shining on an irregularly shaped asteroid induces a net periodic torque that changes its spin rate and obliquity
    • Recently verified by comparing observations with theory
  – Planetary flybys
    • The tidal torques arising from the close passage of an asteroid to a planet can abruptly alter an asteroid’s spin state
    • Large changes in spin state can occur even for non-catastrophic flybys
  – Impacts
    • Sub-catastrophic impacts can impart angular momentum to a body
    • Especially common in the Main Belt
Presumed Dominance of YORP

• As YORP acts on all asteroids continuously, it should be a dominant process for < 10 km-sized asteroids
  – Rotational angular momentum can increase and decrease over time
  – Significant changes in spin state over timescales of 10K - 10M years

• There are a host of interesting questions to ask:
  – How will rubble-pile asteroids respond to this?
  – Can unchecked spin-up cause a body to disrupt into a binary?
  – Under what conditions can these binaries catastrophically disrupt?
  – What is the minimum energy for a catastrophic disruption?
  – What may happen on the way to disruption?

• For some current results, see:

  Rotational fission of contact binary asteroids

Minimum Energy Configurations

- Consider a spinning asteroid with all of its components at rest with respect to each other

- Energy: \[ E = \frac{1}{2} \omega \cdot I_0 \cdot \omega + \mathcal{U}_{00} \]

- Angular momentum magnitude: \[ K = |I_0 \cdot \omega| \]
  
  \( \omega = \) Angular velocity
  
  \( I_0 = \) Inertia matrix of body
  
  \( \mathcal{U}_{00} = \) Self-gravitational potential

\[ \mathcal{U}_{00} = -\frac{G}{2} \int_{B_0} \int_{B_0} \frac{dm_1 dm_2}{|\rho_{12}|} \]

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Minimum Energy Configurations

As spin rate increases or decreases, an aggregate can be placed into a non minimum energy state.

\[ (K_0, E_0) \]

\[ (K_1 = K_0, E_1 < E_0) \]

A perturbation can trigger a shape change, conserving AM, decreasing energy, and dissipating excess energy via friction and seismic waves.
To test this idea, consider the minimum energy configurations of a sphere/ellipsoid system of arbitrary mass fraction $\nu$

$$\nu = \frac{M_{\text{Sphere}}}{M_{\text{Sphere}} + M_{\text{Ellipsoid}}}$$

Minimum energy configuration for $K$ small

Minimum energy configuration for $K$ large
(n,m) definition

n = ellipsoid axis sphere rests on

m = ellipsoid axis system rotates about

(Icarus, in press)
Fission

- If AM continues to grow, the largest components of the system may “fission,” i.e., enter orbit
- Energy and AM can be conserved, but are decomposed:
  - Kinetic Energy
    \[
    \frac{1}{2} \omega \cdot I_0 \cdot \omega = \frac{1}{2} \omega \cdot I_1 \cdot \omega + \frac{1}{2} \omega \cdot I_2 \cdot \omega + \frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} (R \omega)^2
    \]
  - Potential Energy
    \[
    \mathcal{U}_{00} = \mathcal{U}_{11} + \mathcal{U}_{22} + \mathcal{U}_{12}
    \]
  - The mutual potential energy is completely “liberated” and serves as a conduit to transfer rotational and translational KE
$u_{12} = -G \int_{B_1} \int_{B_2} \frac{dm_1 dm_2}{|\rho_{12}|}$
Orbital Evolution

\[ \Delta T_{\text{rot}} + \Delta T_{\text{trans}} + \Delta u_{12} = 0 \]

\[ u_{11} = \text{Constant} \]

\[ u_{22} = \text{Constant} \]
Asteroid Fission

- Rotation periods for fission can be much longer than the surface disruption value of \( \sim 2.5 \) hours.

Value of 1 is orbital rate at the surface of a sphere of given mass.

Two spheres resting on each other will fission at up to twice the period.

If bodies are non-spherical, fission periods are much longer.
Itokawa

Head and Body will orbit at a ~ 6 hour period
Orbit Mechanics after Fission

- The relevant energy for orbital motion is the “free energy,” which is conserved under dynamical evolution:

\[
E_{\text{Free}} = E - \mathcal{U}_{11} - \mathcal{U}_{22}
\]

\[
E_{\text{Free}} = \frac{1}{2} \left[ \omega_1 \cdot I_1 \cdot \omega_1 + \omega_2 \cdot I_2 \cdot \omega_2 + \frac{M_1 M_2}{M_1 + M_2} V \cdot V \right] + \mathcal{U}_{12}
\]

- Energy transfer between orbit and rotation happen rapidly
  - If \( E_{\text{Free}} > 0 \), system can “catastrophically disrupt”
  - If \( E_{\text{Free}} < 0 \), system cannot “catastrophically disrupt”

- Orbits with \( E_{\text{Free}} > 0 \) are highly unstable and usually will send the components away on hyperbolic orbits
Orbital Equilibrium
OE(1,3)

Resting Equilibrium
RE(1,3)
Post-Fission Dynamics

- Orbital stability depends on mass distribution

$1 \times 0.63 \times 0.53$
1 x 0.5 x 0.25
Proto-binaries remain susceptible to disruption if initially unstable

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Increase ellipsoid spin rate by ~2 to cause $E > 0$. 

Eccentricity
Discussion

- Minimum energy for catastrophic disruption of an asteroid
  \[ \omega > \text{Fission limit} \]
  \[ E_{\text{Free}} > 0 \text{ or } \]
  
  Kinetic Energy > - Mutual Potential

\[ \frac{1}{2} I_0 \omega \cdot \omega > -U_{12} \]

- A direct function of how the body is fragmented, or how its mass is distributed
- The fission spin limit is much less than the surface disruption limit, and can approach 2.4 revs/day
- If the body has modest strength, the fission spin rate will be faster and the initial system will have a higher energy, making CD more likely
Conclusions

• A spinning asteroid rotating less than the surface disruption limit may have sufficient energy to undergo catastrophic disruption
  – A contact binary or fractured asteroid can disrupt directly from a relative equilibrium with no additional external energy
• Spin rates as low as ~10 hours can supply sufficient energy for such disruptions to occur
  – Depends on the mass distribution of the body
• The same applies to comets, which should be susceptible to catastrophic disruption at even slower spin rates due to their lower densities