User inputs that affect the motor torque required to drive the OSM.

Enter the desired cycle time. This is the time to move the turret from one position to an adjacent position. The maximum number of cycles for the longest move is 8 cycles.

\[ t_{cycle} = 3.75 \text{ sec} \]

Desired cycle time

Select a desired compression spring for the detent preload force.

The following spring is assumed: Associated Spring Part No. C0360-035-2500-S.

\[ L_{free} = 25 \text{ mm} \quad \text{Free Length of spring} \]

\[ k_{spr} = 2.97 \frac{N}{\text{mm}} \]

Select a desired preload force for seating the detent into the vee-groove.

\[ F_{pre} = 15N \quad \text{Desired initial preload of compression spring when the detent is seated into the vee-groove.} \]

The following two stepper motors are considered for driving the turret.

Motor Parameters for Phytron Steppers

\[ T_{vss42\_max} = 92N \text{ mm} \quad \text{VSS42 Stepper motor drive (stall) torque} \]

\[ T_{vss32\_max} = 32N \text{ mm} \quad \text{VSS 32.200.1.2-VGPL 32 4-UHVC stepper motor (with 4:1 reduction gear box (stall) torque} \]

\[ \text{Ratio}_{motor\_gearbox} = 1 \]

Select a desired gear reduction ratio between the motor and the crank of the geneva mechanism (default ratio is 3:1).

\[ \text{Ratio}_{mtergare} = 3 \quad \text{Miter gear reduction ratio between stepper and crank} \]

Define crank shaft bearing friction (torque).

\[ T_{crank\_friction} = 1N \text{ mm} \]

Define motor drive shaft bearing friction (torque).

\[ T_{fric\_shaft1} = T_{crank\_friction} \]

Define OSM hub bearing friction (torque).

\[ T_{fric\_fric} = 10N \text{ mm} \]
Part 1. Cam Move Profile Calculations

This calculation assumes a trapezoidal move profile for the Geneva crank. The crank starts at $\theta_1 = 0$ deg, which corresponds to the detent mechanism fully seated in the vee-groove. The crank accelerates from $\theta_1$ to $\theta_2$ with constant acceleration. From $\theta_2$ to $\theta_3$, the angular velocity is constant. From $\theta_3$ to $\theta_4$, the crank decelerates back to a stop, completing one cycle of the crank.

The geometry of the Geneva wheel will determine at which angle the cam drive will engage the geneva:

- $R_{CG}$: Radius of the cam drive axis to the axis of the Geneva wheel
- $R_G$: Radius of the cam axis to the drive bearing axis
- $N_S$: Number of discrete positions

$$\theta_{engage} = \frac{\pi}{2} \tan\left(-\frac{R_{CG} - R_G \cos\left(\frac{\pi}{N_S}\right)}{R_G \sin\left(\frac{\pi}{N_S}\right)}\right) = 111.898 \text{ deg}$$

Angle at which the drive engages the geneva

$$R_{CD} = \sqrt{(R_{CG} - R_G \cos\left(\frac{\pi}{N_S}\right))^2 + (R_G \sin\left(\frac{\pi}{N_S}\right))^2} = 15.487 \text{ mm}$$

Distance from the cam axis to the drive bearing axis

$\theta_0 = 0$ deg

Starting position of the crank, which corresponds to the detent mechanism seated in the vee-groove.

$\theta_1 = 10$ deg

Cam angle when follower begins to lift detent off of vee-groove.

$\theta_2 = \theta_{engage} = 111.898 \text{ deg}$

Position of crank when the detent mechanism completely clear the vee-groove. The crank accelerates to full speed from $\theta_1$ to this position and begin to engage with the geneva wheel to begin cycling the turret to the next position.

$\theta_3 = 360$ deg

Position of crank when the geneva has completed a move of the turret to the next position. The crank rotates at constant speed from this position to $\theta_4$.

$\theta_4 = 360$ deg

Cam angle when follower finishes seating the detent onto the vee-groove.

$\theta_5 = 360$ deg

Ending position of the crank, which corresponds to the detent mechanism seated in the vee-groove.

$\theta = \theta_5, 1$ deg... $\theta_5$

Define range for crank angle.

Calculate the corresponding times for the angular positions above.

$$\omega_{crank\_max} \geq \frac{1}{\text{cycle}}(20\theta_5 - \theta_1 + \theta_2) = 25.947 \text{ rev/min}$$

Calculated maximum crank velocity for the desired cycle time

$$t_2 = 2 \cdot \frac{\theta_2}{\omega_{crank\_max}} = 1.438 \text{ s}$$

Calculated time to get to the $\theta_2$ position.

$$t_3 = \frac{2\theta_4}{\omega_{crank\_max}} = 0.43 \text{ s}$$

Calculated time to get to the $\theta_3$ position.

$$t_5 = t_4 + 2 \cdot \frac{\theta_4}{\omega_{crank\_max}} = 3.75 \text{ s}$$

Calculated time to get to the $\theta_5$ position.

$$t_4 = t_5 - t_3 = 3.32 \text{ s}$$

Calculate angular position of crank as a function of time.

$$t := \theta_5, 0.01 \ldots t_5$$

Define time range

$$\theta_k(t) := \begin{cases} t < t_2 & \frac{\omega_{crank\_max}^2}{2} t^2 + \omega_{crank\_max} t_2 + \omega_{crank\_max} (t - t_2) \\ t \geq t_2 & \omega_{crank\_max} (t - t_2) \end{cases}$$

$$\theta_{crank}(t) := \begin{cases} t < t_3, \theta_k(t) & \frac{\omega_{crank\_max}}{t_5 - t_3} \left[t_5^2 t - \frac{1}{2} t^2\right] + \theta_5 - \frac{\omega_{crank\_max}}{t_5 - t_3} \frac{t_5^2}{2} \\ t \geq t_3 & \theta_k(t) \end{cases}$$

Calculate angular velocity of crank as a function of time.

$$\omega(t) := \begin{cases} t < t_2 & \omega_{crank\_max} \frac{t}{t_2} \\ t \geq t_2 & \omega_{crank\_max} \end{cases}$$

$$\omega_{crank}(t) := \begin{cases} t < t_2 & \omega(t) \\ t \geq t_2 & \omega_{crank\_max} \frac{t_5 - t}{t_5 - t_3} \end{cases}$$
Calculate angular acceleration of crank as a function of time.

\[ \alpha_{\text{crank}}(t) = \frac{d}{dt} \omega_{\text{crank}}(t) \]  

**Crank Angular Acceleration**

**Crank Angular Position**

**Crank Angular Velocity**

Angle of the geneva in function of the angle of the cam:

\[
\gamma_{\text{geneva}}(t) = \arctan \left( \frac{R_{\text{CD}} \cos \left( \theta_{\text{crank}}(t) - \frac{\pi}{2} \right)}{R_{\text{CG}} R_{\text{CD}} \sin \left( \theta_{\text{crank}}(t) - \frac{\pi}{2} \right) - \frac{\pi}{2}} \right)
\]
This calculation uses a sinusoidal cam follower path for a smooth transition between the seated to unseated positions of the detent mechanism.

Lift = 7mm  
Cam lift (distance from detent fully seated to fully disengaged).

R₂ = 15 mm  
Radius of cam when detent is fully disengaged.

R₁ = R₂ - Lift  
Radius of cam when detent is seated.

Define cam radius equation for three segments of cam from 0 deg to θ₂ & θ₃ to 180 deg.

\[ R_{zone1}(θ) = \begin{cases} \frac{R_2 - R_1}{2} [1 + \sin \left( \frac{180 deg}{\theta_2 - \theta_1} \right)(θ - \theta_1) - 90 deg] \end{cases} \]

Radius of cam path in zone 1.

\[ R_{zone12}(θ) = if(θ < \theta_2, R_{zone1}(θ), R_2) \]

Radius of cam path in zones 1 & 2.

\[ R_{zone123}(θ) = \begin{cases} \frac{R_2 - R_1}{2} [1 + \sin \left( \frac{180 deg}{\theta_4 - \theta_3} \right)(θ - \theta_3) + 90 deg] \end{cases} \]

Radius of cam path in zones 1, 2 & 3.

\[ R_{zone1234}(θ) = if(θ < \theta_4, R_{zone123}(θ), R_1) \]

Radius of cam path in zones 1, 2, 3 & 4.

\[ R_{cam}(θ) = R_{zone1234}(θ) \]

Follower Path vs. Follower Angle

Cam Follower Path Calculations

Cam Follower Path

Follower Path
Convert cam follower path to cartesian coordinates

\[ x(t) = R_{\text{cam}}(\theta(t)) \cos(\theta(t)) \]
\[ y(t) = R_{\text{cam}}(\theta(t)) \sin(\theta(t)) \]

Calculate angle \( \eta \):
\[ \eta(t) = \theta_{\text{crank}}(t) - \beta(t) \]

Calculate moment arm \( BB \):
\[ BB(t) = R_{\text{cam}}(\theta(t)) \sin(\eta(t)) \]

Motor & Drive Inertias
\[ I_{\text{Vss32}} = 1000 \text{gm mm}^2 \]
\[ I_{\text{shaft1}} = 2218 \text{gm mm}^2 \]
\[ I_{\text{crank}} = 8763 \text{gm mm}^2 \]
\[ I_{\text{ftw}} = 1769112 \text{gm mm}^2 \]
\[ I_{\text{mot gear}} = 0 \]
Calculate loads on crank in Zone 1 & Zone 3.
(Cam angle from 0 degrees to \(\theta_2\) and \(\theta_3\) to \(\theta_5\)).

The crank is subjected to the following loads in zones 1 & 3.
1. Flex Pivot loads from detent mechanism.
2. Preload force from detent mechanism compression spring.
3. Inertial forces caused by accelerating mass of detent mechanism when crank is moving.
4. Inertial force (torque) on crank shaft from accelerating crank (I \(x\) \(\omega\)).
5. Frictional forces (torque) from bearings.

1. Calculate Flex Pivot Load.

\[
L_{\text{arm}} = 100.45085 \text{mm}
\]

\[
\alpha_{\text{detent}} := \arcsin \left( \frac{R_{\text{cm}}(\theta_1) - R_1}{L_{\text{arm}}} \right)
\]

\[
k_f = 0.0286 \text{in-lbf/deg}
\]

\[
F_{\text{fp\_at\_cam}}(t) = k_f \frac{\alpha_{\text{detent}}}{L_{\text{arm}}}
\]

Component of detent spring force provided by the parallel spring flexure.

2. Calculate detent mechanism preload compression spring forces.

\[
L_{\text{arm\_at\_detent}} = 60 \text{mm}
\]

\[
L_{\text{arm\_at\_spring}} = 48.8 \text{mm}
\]

\[
F_{\text{pre}} = \frac{L_{\text{arm\_at\_detent}}}{k_{\text{spr}}} = 18.687 \text{ mm}
\]

Length at initial preload (seated in vee-groove).

\[
L_{\text{maxcomp}} = L_{\text{preload\_at\_detent}} - \frac{L_{\text{arm\_at\_detent}}}{L_{\text{arm\_at\_spring}}} = 15.342 \text{ mm}
\]

Length at max compression (unseated)

\[
F_{\text{comp}}(t) = F_{\text{pre}} \frac{L_{\text{arm\_at\_detent}}}{L_{\text{arm}} + k_{\text{spr}}(R_{\text{cm}}(\theta_1) - R_1) - L_{\text{arm\_at\_spring}}}
\]

Component of detent spring force provided by the compression spring.

3. Calculate the inertial forces caused by accelerating mass of detent mechanism when crank is moving.

\[
v_{\text{detent}}(t) := \frac{\frac{d}{dt}R_{\text{cm}}(\theta_1)}{R_{\text{cm}}(\theta_1)}
\]

Linear velocity of detent arm when climbing the cam.

\[
a_{\text{detent}}(t) := \frac{\frac{d^2}{dt^2}R_{\text{cm}}(\theta_1)}{R_{\text{cm}}(\theta_1)}
\]

Linear acceleration of detent arm when climbing the cam.

\[
M_{\text{detent}} = 42.75 \text{ gm}
\]

Mass of moving portion of detent arm.

\[
F_{\text{detent\_inertial}}(t) = M_{\text{detent}} a_{\text{detent}}(t)
\]

Force component of detent arm due to the acceleration when climbing cam.

Calculate total force on cam from follower loads.

\[
F_{\text{follower}}(t) = F_{\text{comp}}(t) + F_{\text{fp\_at\_cam}}(t) + F_{\text{detent\_inertial}}(t)
\]

Total detent spring force.

\[
F_{\text{normal}}(t) = \frac{F_{\text{follower}}(t)}{cos(\eta(t))}
\]

Reaction force to the spring in the normal direction

Cam Follower Forces vs. Time

Calculate motor speed and acceleration

\[
\omega_{\text{shaft}}(t) := \frac{\omega_{\text{crank}}(t)}{\text{Ratio\_mitergear}}
\]

\[
\omega_{\text{motor}}(t) := \frac{\omega_{\text{shaft}}(t)}{\text{Ratio\_motorbox}}
\]
Calculate loads on crank in Zone 2
(Cam angle from $\theta_2$ to $\theta_3$).

The crank is subjected to the following loads in zone 2.

1. Inertial force (torque) of accelerating turret (I x $\alpha$) as is cycles from one position to the next.
2. Frictional forces (torque) from turret bearings.

- **R_crank** = 13.55826 mm
- **D_ctr** = 64 mm

\[ \theta_{gen2}(t) = \text{atan}\left(\frac{R_crank \sin(180\deg - \theta_crank(t))}{D_ctr - R_crank \cos(180\deg - \theta_crank(t))}\right) \]

\[ \theta_{geneva}(t) = \text{atan}\left(\frac{R_crank \sin(180\deg - \theta_crank(t))}{D_ctr - R_crank \cos(180\deg - \theta_crank(t))}\right) \]

\[ R_{geneva}(t) = \frac{R_crank \sin(180\deg - \theta_crank(t))}{\sin(\theta_{geneva}(t))} \]

\[ \omega_{geneva}(t) = \frac{d\theta_{geneva}(t)}{dt}, \quad \alpha_{geneva}(t) = \frac{d^2\theta_{geneva}(t)}{dt^2} \]

\[ T_{ftw}(t) = I_{ftw} \omega_{geneva}(t) + T_{fric} \]

\[ F_{tan_turret}(t) = \begin{cases} \frac{T_{ftw}(t)}{R_{geneva}(t)} & \text{if } t < t_2 - 0,00 \text{hr} \\ 0 & \text{otherwise} \end{cases} \]

\[ F_{tan_crank}(t) = F_{tan_turret}(t) \sin(\theta_{c_rank}(t) - \theta_{geneva}(t) - 90\deg) \]

\[ T_{crank_z2}(t) = F_{tan_crank}(t) R_{crank} \]

\[ T_{normal}(t) = I_{crank} \alpha_{crank}(t) + T_{c_rank_fric} + F_{tan_crank}(t) R_{c_rank} \]

\[ T_{shaft}(t) = I_{shaft} + I_{mot_gearbox} \alpha_{shaft}(t) = \frac{T_{shaft}(t) \text{Ratio}_{nitingear}}{\text{Ratio}_{mot_gearbox}} + T_{fric_shaft} \]

\[ T_{mot_gearbox}(t) = I_{vs32} \alpha_{mot}(t) + \frac{T_{shaft}(t)}{\text{Ratio}_{mot_gearbox}} \]

\[ T_{mot_nogrbx}(t) = I_{vs32} \alpha_{shaft}(t) + T_{shaft}(t) \]
Drive Loads (with motor gearbox)
Drive Loads (without motor gearbox)
VSS 32 stepper max torque
VSS 42 stepper max torque
Detent Ball Contact, Deformations and Friction:

\[ F_{\text{detball}} = 0, 1, N, 500N \]

\[ R_{\text{ball}} = 6\text{mm} \quad \text{Detent Ball Radius} \]

\[ E_{\text{ball}} = 2 \cdot 10^{11}\text{Pa} \quad \text{Detent Ball Young's Modulus} \]

\[ \nu_{\text{ball}} = 0.3 \quad \text{Detent Ball Poisson's Coefficient} \]

\[ E_{\text{vgroove}} = 1 \cdot 10^{11}\text{Pa} \quad \text{V-Groove Young's Modulus} \]

\[ \nu_{\text{vgroove}} = 0.33 \quad \text{V-Groove Poisson's Coefficient} \]

\[ \frac{dF_{\text{detball}}}{R_{\text{ball}}} = \left[ \frac{2}{3} \left( 1 - \nu_{\text{vgroove}}^2 \right) E_{\text{vgroove}} \left( 1 - \nu_{\text{ball}}^2 \right) E_{\text{ball}} - \frac{F_{\text{detball}}}{R_{\text{ball}}} \right]^{\frac{2}{3}} \]

**Sphere - Elastic 1/2 space Contact**

**Ball / V-groove friction**:

Static Friction Coefficients:

\[ \mu_{\text{sas}} = 0.61 \quad \text{Aluminum / Steel} \]

\[ \mu_{\text{sbs}} = 0.51 \quad \text{Brass / Steel} \]

\[ \mu_{\text{ss}} = 0.8 \quad \text{Steel / Steel} \]

V-groove angle:

\[ \alpha_{\text{vgroove}} = 45\text{deg} \]

\[ F_{\text{fas}} = 100 \cdot \cos \left( \frac{\pi}{4} - \alpha_{\text{vgroove}} \right) \mu_{\text{sas}} = 43.134 \]